Simultaneous Propagation of Bound and Leaky Dominant Modes on Printed-Circuit Lines: A New General Effect

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***Abstract*—We were the first to report [1] that both the bound and leaky dominant modes can propagate *simultaneously* on conductor-backed coplanar strips over a frequency range. We have recently studied this interesting and initially unexpected effect in more detail, and we have made two important discoveries: First, the simultaneous-propagation effect can actually occur on *most*, if not all, printed-circuit transmission lines (its presence depending on the relative line dimensions), so that, contrary to earlier belief, the effect is rather *general*. Second, we have discovered the surprising presence of a *new improper* (or nonspectral) *real solu tion*, which is *nonphysical* but whose evolution as a function of dimensional change serves to *explain* how the simultaneous-propagation effect can occur. The new solution, and its behavior in a completely nonphysical region, thus governs otherwise-mysterious large changes in the physical, measurable solutions.**

I. Introduction

LESS THAN A DECADE ago, everyone thought that the dominant mode on printed-circuit transmission lines was purely bound at all frequencies. We now know that most transmission lines of this type are purely bound only at low frequencies. As the propagation frequency becomes *greater* than some critical value, the bound mode turns into a *leaky mode*, with power leaking away at some angle in the form of a surface wave on the surrounding substrate, or a parallel-plate mode if there is a top cover over the substrate. The precise type of surface wave depends on the specific structure. In certain cases the situation can become more involved. For example, for microstrip line on an isotropic substrate, the well-known dominant mode does remain bound at all frequencies, but, above some critical frequency, a second dominant mode (with the same current distribution and field near the strip as the first dominant mode) is found to be present, and it is *leaky*. On the other hand, if the substrate has the right type of anisotropy, then the first dominant mode also becomes leaky above some critical frequency, but not at the same frequency. Furthermore, when slot line or coplanar waveguide has strips of infinite width and is conductor-backed, the dominant mode is leaky at *all* frequencies. It is of great practical importance whether or not the transmission-line dominant mode is leaky, because such leakage can not only produce power loss from the line but also introduce unexpected cross talk between neighboring portions of a microwave or millimeter-wave integrated circuit, and thereby degrade the performance of the circuit.

For most printed-circuit lines, however, the guided dominant mode is purely bound at lower frequencies, and then becomes leaky above some critical frequency. This critical frequency occurs when the dispersion curve for the dominant mode crosses the dispersion curve for the surface wave (or parallel-plate mode) into which the leakage occurs. At this crossing, we always find some complicated fine structure and a small range in frequency within which the solution for the dominant mode is *nonphysical.* We have called this small range a "spectral gap." In the usual situation, therefore, we find that the bound dominant mode can be observed only *below* the frequency at which the spectral gap sets in, and the leaky dominant mode propagates only *above* the frequency at which the spectral gap ends, so that the two solutions are *completely separated* from each other.

Until recently, everyone thought that when the *relative dimensions* of the cross section were changed, the only effects would be to modify the values of the propagation wavenumber and the characteristic impedance. We now know that this assumption is also incorrect.

We were the first to report [1] that under certain circumstances the dispersion curve for the dominant mode can display behavior quite different from that described above. We discovered that there can exist a frequency range within which the bound and leaky modes can propagate *simultaneously*, and that the spectral gap then disappears. Furthermore, the simultaneous propagation was produced by changing *only* the relative cross-sectional dimensions of the transmission line. We also found that the frequency range over which the simultaneous propagation occurs can in fact become quite large.

This phenomenon was found first for conductor-backed coplanar strips [1], and our initial view was that the effect was unique to that transmission line, or perhaps others very similar to it. The *first* major conclusion revealed by our study since then is that the simultaneous-propagation effect is actually quite *general*, rather than being unique or rare. We have now found that this effect also occurs for other lines such as slot line and unbacked coplanar strips, on isotropic

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or anisotropic substrates, and for strips with symmetrical or unsymmetrical widths. We also found it for microstrip line on anisotropic substrates. A recent paper [2], following [1], demonstrated similar behavior for conductor-backed coupled slot lines, where the pair of slots may be viewed as a sort of dual to the pair of strips in our case of conductor-backed coplanar strips.

The usual behavior, where the bound and leaky solutions are completely separated from each other in frequency, is found when, for coplanar strips, the strip widths are relatively narrow, and for slot lines, the slot spacing is relatively narrow. When the strip widths and the slot spacing, respectively, are *increased* sufficiently, the propagation behavior changes systematically into the regime in which the bound and leaky ranges overlap, so that we find the bound mode and the leaky mode propagating *simultaneously* within the same frequency range. In Section II of this paper, we describe this effect in more detail, and then demonstrate the dramatic differences in dispersion behavior that occur as the strip widths (or slot spacings) are changed from narrow to wide. The results are presented first for conductor-backed coplanar strips, and then for unbacked single slot line, as examples of rather different transmission-line structures, to illustrate the *generality* of this new effect. *Measurements* of the phase and leakage constants were made for each case for each of these lines, following the method described in [3], and the results of these measurements are also compared with the calculated values in Section II.

The *explanation* for the transition from the usual case, for which the bound and leaky solutions are separated, to the case for which simultaneous propagation can occur, remained obscure, however. The *second* major contribution from our recent study is the discovery of a *new improper* (nonspectral) *real* solution, which is clearly nonphysical but which serves to explain how this transition can occur. An examination of the evolution of this new solution in the plot of propagation wavenumber versus frequency, as the relative transmission-line dimensions are modified, shows that the complicated changes that occur in the nonphysical region ultimately lead to important and dramatic changes in the *physical* region. Section III discusses the properties of the new solution and shows how its evolution leads to the phenomenon of simultaneous propagation.

The *vector electric field plots* for the different types of solutions are presented and discussed in Section IV for slot line. Of particular interest is the field behavior for the new improper real solution, which is different from anything encountered previously.

The two major aspects of this study, the *generality* and the *explanation*, are therefore considered separately in Sections II and III, respectively.

II. The Simultaneous-Propagation Effect and Its Generality

As indicated in the Introduction, this effect was first found in connection with conductor-backed coplanar strips [1]. We were examining the effect of increasing the width of the strips, and we noted to our surprise that the nature of the dispersion plot (normalized phase constant β/k_0 versus normalized frequency h/λ_0) for the dominant mode became altered in an unusual way. When the strips were made sufficiently wide (although not really very wide), we found that the spectral gap disappeared entirely, and that the bound-mode and leaky-mode portions were no longer separated from each other but were present *simultaneously* within the same frequency range. This frequency range can actually be very large; for example (as can be seen later in Fig. 4), for $w/h = 0.70$, where w and h are the strip width and substrate height, respectively, the bound and leaky modes can propagate simultaneously over an octave in frequency.

The bound mode and the leaky mode, whether separated in frequency or present simultaneously in the same frequency range, have very similar current distributions on the strips and field distributions near the central portion of the line. Thus, when the two dominant modes are present at the same frequency and one tries to excite the bound mode, the leaky mode will be excited at the same time and with roughly the same amplitude. If the circuit is designed on the assumption that only the bound mode is present but unexpectedly a leaky mode is there as well, the circuit will then experience unwanted and unexpected cross talk and power loss. The simultaneous-propagation effect described here is thus not only an intriguing propagation phenomenon in its own right but it has important *practical* consequences as well.

The practical implications become even more significant when it is realized, as we show here, that the simultaneous-propagation effect is not a rarity, as we thought originally, but is quite *general.*

The second transmission line that exhibited this effect to us was microstrip line on a boron nitride substrate. An earlier analysis [4] of dominant-mode propagation on microstrip line placed on an anisotropic substrate employed Epsilam-10 as the substrate, and we found then that the bound-mode and leaky-mode portions were separated in what we have called the usual fashion. When the substrate was changed to boron nitride, however, with the line dimensions remaining the same, we found that simultaneous propagation occurred. We believe that the difference in behavior is due to the greater anisotropy ratio for boron nitride. Then we returned to the Epsilam-10 substrate and further increased the strip width of the microstrip line, and we indeed found that simultaneous propagation occurred there as well.

At that stage we suspected that the effect may be a general one, occurring on most, if not all, types of printed-circuit lines. We therefore varied the relative dimensions in the cross sections of a number of other lines, such as unbacked slot line (with infinite side strips) and unbacked coplanar strips. Then we varied only one of the side strips on coplanar strips, making the structure asymmetrical. Finally, we employed anisotropic substrates with some of these structures. In *all cases* we were able to find a range of strip widths or slot widths for which the bound mode and the leaky mode were capable of propagating simultaneously over a frequency range.

For two of these transmission lines we also took *measurements* over a wide frequency range of both the normalized phase constant β/k_0 and the normalized leakage constant

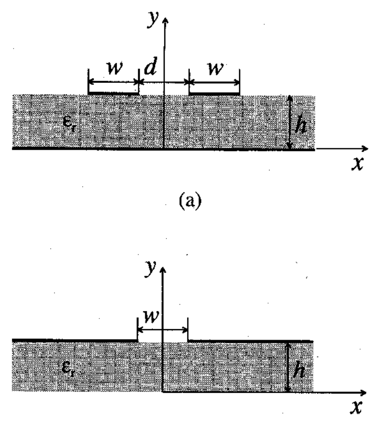

Fig. 1. The two printed-circuit transmission lines for which numerical results are presented in this paper. (a) Conductor-backed coplanar strips, (b) Slot line.

α/k_0, first for a set of dimensions for which the bound mode and the leaky mode are present completely separated in frequency, and then for a set of dimensions for which these two modes are present simultaneously over a frequency range. The details of the measurement procedures employed are presented in [3]. The measurements were taken to verify that simultaneous propagation can actually occur.

The two transmission-line types for which measurements were taken are conductor-backed coplanar strips and unbacked single slot line with infinite metal sides. Since it is not practical to include theoretical results here for a large variety of printed-circuit lines in order to justify the claim of generality for the simultaneous-propagation effect, we have instead chosen to present results for only those two lines since we can then compare the theoretical calculations with these measured values. The cross sections of these two transmission-line types are shown in Fig. 1. The theoretical approach employed in all the calculations was the spectral-domain method, with the path of integration appropriately deformed to enclose the poles of the surface wave into which the leakage occurs. When the mode is purely bound, the integration path is taken, following the standard procedure, along the real axis in the transverse wavenumber (k_x) plane. When the mode is leaky, and leakage occurs in surface-wave form, it is necessary to include in addition the contribution from the surface-wave poles, which now lie on the improper portion of the k_x plane. These poles are captured by deforming the integration path [5], as mentioned above.

A. Conductor-Backed Coplanar Strips

We select conductor-backed coplanar strips for the first example that demonstrates that the propagation behavior can be changed in a dramatic way by altering only the relative dimensions of the line cross section. The structure is shown in Fig. 1(a), which also indicates the notation for the various dimensions. When we vary the relative dimensions in the line's cross section we maintain $\varepsilon_r = 2.25$ and $d/h = 0.25$, and we modify the value of w/h.

We first consider the case of *narrow* strip widths w, and select $w/h = 0.25$, for which Fig. 2 applies. We observe

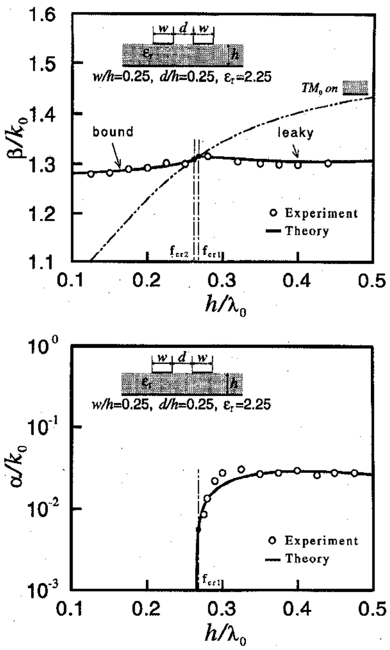

Fig. 2. Comparisons of theoretical and measured values of the normalized phase constant β/k_0 and leakage constant α/k_0 for conductor-backed coplanar strips, as a function of normalized frequency, h/λ_0, for the case of *narrow* strips, $w/h = 0.25$. The bound and leaky modes are separated from each other by the spectral gap, which is present between frequencies f_{cr2} and f_{cr1}.

in Fig. 2 that the bound and leaky portions of the dominant mode are clearly *separated* from each other, which is the usual and expected situation. The pattern of the spectral gap between the bound and leaky portions is of the standard form, although it is difficult to observe it in Fig. 2 because of the compressed scale. The spectral gap actually appears between the vertical lines in the β/k_0 plot labeled f_{cr2} and f_{cr1}, where f_{cr2} and f_{cr1} indicate, respectively, the frequencies at which the bound mode ends and the physical leaky mode begins. The circles represent the measured values, which are seen to agree reasonably well with the theoretical calculations.

When the relative strip width is *increased* to $w/h = 0.50$, the resulting propagation behavior becomes that shown in Fig. 3, where the spectral gap has disappeared and the bound and leaky modes are both present *simultaneously* in the frequency range between f_{cr1} and f_{cr2}. We note that f_{cr2} now occurs at a higher frequency than f_{cr1}, in contrast to the situation in Fig. 2, so that an overlap region is formed. As in Fig. 2, the circles represent the measured values, which again agree well with the theoretical values. The theoretical curves extend beyond the regions shown, as will be explained later, but the solutions are nonphysical in those regions and are therefore omitted here.

When the relative strip widths are increased further, the overlap region between f_{cr1} and f_{cr2} also increases. In Fig. 4 the value of w/h is increased to 0.70, from 0.50 in Fig. 3, which is not a large increase, yet the frequency range within which the bound and leaky modes are both present simultaneously is now over an octave wide. We should also note that the leaky mode now starts at a significantly lower frequency, so that the problems related to cross talk must be faced at lower frequencies.

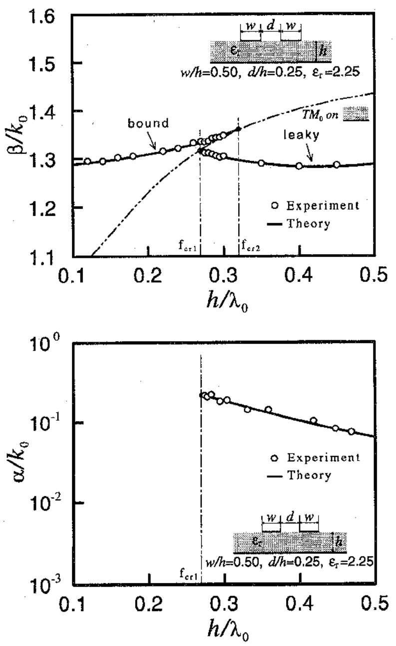

Fig. 3. Comparisons similar to those in Fig. 2, but for *wider* strips, $w/h = 0.50$. The spectral gap has now disappeared, and the bound and leaky modes are both present simultaneously in the frequency range between f_{cr1} and f_{cr2}.

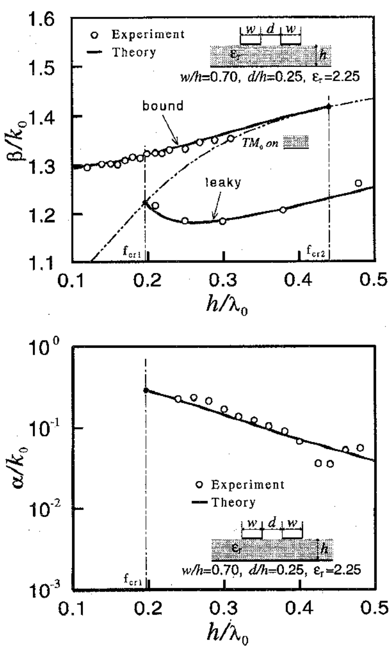

Fig. 4. Comparisons similar to those in Fig. 2, but for strips that are somewhat wider ($w/h = 0.70$) than those in Fig. 3. The frequency range for simultaneous propagation is now over an octave wide.

We may therefore draw two basic conclusions:

1) For *narrow* strips, the bound and leaky portions of the dominant mode are separated from each other, with a spectral gap between them, whereas for *wide* strips the spectral gap disappears and the bound and leaky modes are both present simultaneously within some frequency range.

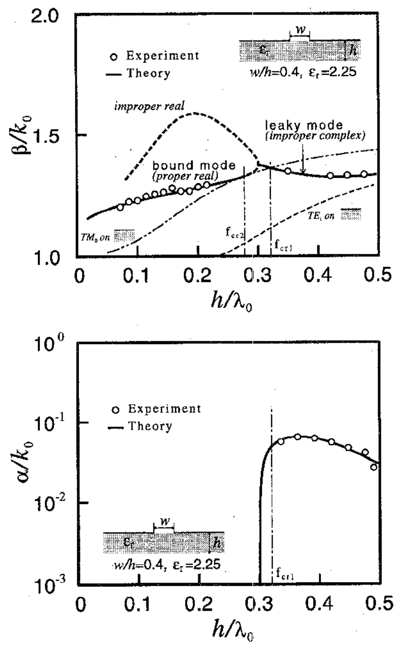

Fig. 5. Comparisons similar to those in Fig. 2, but for *slot line*, and for a *narrow* slot width, $w/h = 0.40$. The bound and leaky modes are separated, as in Fig. 2, with the spectral gap between them more pronounced than in Fig. 2.

2) Within the simultaneous-propagation regime, wider strips will produce a wider frequency range within which simultaneous propagation can occur, and will cause the leaky mode to begin at a lower frequency.

B. Slot Line

Conventional slot line (with a single slot, unbacked, and with infinitely wide metal side strips) is selected as the second example in order to show that similar results can occur on a type of line quite different from the conductor-backed coplanar strips discussed under Section II-A. The line cross section, and the notation used for the dimensions, are presented in Fig. 1(b). In Section II-A, w represented the width of the coplanar strips, but here w indicates the width of the slot.

We begin with a relatively *narrow* slot, with $w/h = 0.40$ and $\varepsilon_r = 2.25$, where w, as stated above, is the slot width, and h is the height of the dielectric substrate layer. The dispersion results for this case are shown in Fig. 5, and it is seen that the behavior is basically similar to that displayed in Fig. 2, where the bound and leaky modes are *completely separated* from each other. Again, we observe that the measured values and the theoretical values agree well with each other.

Three interesting differences between the curves in Fig. 5 and in Fig. 2 are that the onset of leakage for the slot line (Fig. 5) occurs at a somewhat higher relative frequency, that the maximum value for the leakage constant is higher for the slot line, and that the spectral gap is wider for the slot line (usually associated with a higher leakage rate). In fact, the spectral gap, between the vertical lines for f_{cr2} and f_{cr1}, is sufficiently wide in Fig. 5 that we can now discern the composition of the spectral gap. It is seen first that the portion at lower frequencies, which is the continuation

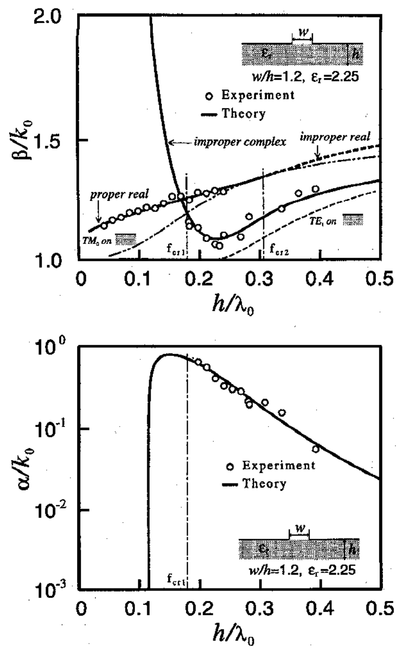

Fig. 6. Comparisons similar to those in Fig. 2, but for *slot line*, and for a *wide* slot, $w/h = 1.2$. As in Figs. 3 and 4, the spectral gap now disappeared, and the bound and leaky modes can propagate simultaneously between f_{cr1} and f_{cr2}.

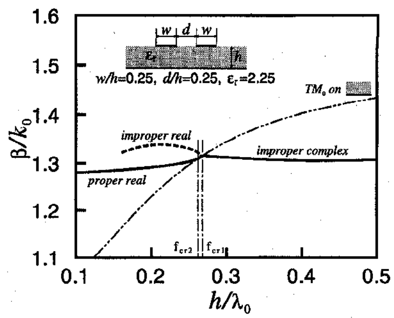

Fig. 7. The normalized phase constant β/k_0 for *narrow* strips, $w/h = 0.25$, as a function of normalized frequency h/λ_0, for conductor-backed coplanar strips. The bound and leaky modes are separated, with a spectral gap between them, and the continuation of the improper real solution which doubles back to lower frequencies is also shown.

of the bound mode, is shown dotted, and that it is also shown continuing and curving back to lower frequencies. This solution is an improper (or nonspectral) real solution, which is nonphysical and ordinarily has no meaning. That is why the corresponding portion has not been shown in Fig. 2. As will be seen in Section III, however, this nonphysical solution plays an important role in the transformation to the simultaneous-propagation case as the slot width is increased gradually. The solid line portion inside the spectral gap, which is at the higher frequency side and connects to the physical leaky mode, is also a leaky solution, but with very limited physical meaning.

As we increase the ratio w/h, *widening* the slot, the spectral gap changes its nature, so that when we finally reach $w/h = 1.2$, as in Fig. 6, the dispersion behavior becomes completely different. We note in Fig. 6, as in Figs. 3 and 4, that the spectral gap has now disappeared, and that the bound and leaky modes are now present *simultaneously* over a fairly wide frequency range.

We should also recognize that in the region of simultaneous propagation, between f_{cr1} and f_{cr2}, the value of α/k_0 is very large, representing a very rapid rate of power decay, which varies roughly from 10 dB per wavelength to 30 dB per wavelength. Two conclusions should be drawn from this result. First, it becomes very difficult to measure accurately the values of β/k_0 and α/k_0 for the leaky mode in this frequency range, and this difficulty accounts for the scatter present in the measured data. The second point relates to the *physical interpretation* of the result. A leaky mode with a mild decay rate along the line will leak power at some specific angle from the axis of the line. If the decay rate is enormous, the power is essentially spurted out at all angles from the excitation point. In the present case, there is no doubt that the leaky mode will be excited within the circuit, but the physical effect of this excitation should more closely resemble that of a strongly radiating discontinuity than that of a leaky transmission mode in the usual sense. In any case, problems involving cross talk and power loss will certainly be present.

The behavior shown in Fig. 6 has not been known previously for slot lines, and it may come as a surprise to many. We should note, however, that it occurs only for relatively wide slots, wider than those customarily employed. It is important to recognize, furthermore, that such very different dispersion behavior can result from modifying *only* the relative dimensions of the line's cross section.

III. The New Improper Real Solution: Its Evolution and Role in Explaining the Simultaneous-Propagation Effect

The discussion so far has described the simultaneous-propagation effect, has shown that it results from changing only the relative dimensions of the line's cross section, and has stressed that the effect is general, rather than being restricted to only one or two line types. What is still missing from the discussion is some sort of *explanation* as to what else is happening to permit this mysterious transition from separate frequency regions for the bound and leaky modes to a frequency range within which these two modes can propagate simultaneously. In this section, we examine the world of *nonphysical* solutions, and follow their evolution and change as a function of the line's dimensional changes. We will see that the changes in these *nonphysical* solutions, which we would ordinarily ignore, govern the otherwise-mysterious large changes in the physical, measurable solutions discussed in Section II.

A. A Puzzle Regarding the Number of Solutions

Let us first look at Fig. 7, which holds for conductor-backed coplanar strips, and applies to narrow strips, for which the bound and leaky modes are completely separated. This figure differs from that for β/k_0 in Fig. 2 in that the measurements

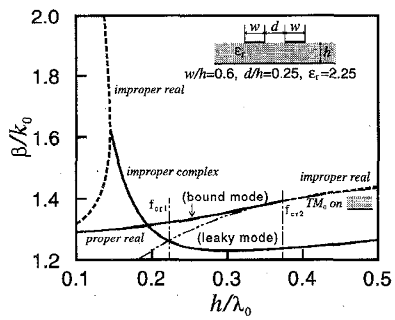

Fig. 8. A plot similar to that in Fig. 7 but for *wider* strips, $w/h = 0.60$, but the plot now looks totally different. The spectral gap has disappeared, the bound and leaky modes can now propagate simultaneously between $f_{\mathrm{cr}1}$ and $f_{\mathrm{cr}2}$, and new improper real solutions are present.

are not included, and, more important, that the improper solution has been added. In the bound-mode region, we see that there are two solutions, the actual bound mode, which is real and proper, and an improper real mode, which is nonphysical. In the leaky-mode region, we have the leaky mode and also its complex conjugate, which is nonphysical (since its amplitude increases as the wave propagates) and also has the same value of β as the leaky mode so that we cannot see it separately on the dispersion plot. Thus, we have *two* solutions at *all* frequencies.

When we look at Fig. 8, however, for which the bound and leaky modes are present simultaneously over a frequency range, we notice that there are *three* solutions at all frequencies. Again, we must remember that the leaky solution has its complex conjugate, and notice that for very low frequencies there are now two improper real solutions. This change in the number of solutions appears mysterious, and it contradicts what we expect from analytical continuity; a solution in Fig. 7 should migrate continuously to a corresponding one in Fig. 8 at each frequency, as the w/h value is increased continuously. However, we do not find that this is the case when we inspect these two figures.

To explain this mysterious behavior, we intuitively presumed that there should be a solution that is presently *missing* at all frequencies in Fig. 7. We therefore searched for such a new solution in the region of $\beta/k_0 > [\varepsilon_r]^{1/2}$ when $w/h = 0.25$, and found, as we expected, a *new improper real* solution which lies in the region $\beta/k_0 > 1.5$ in the frequency range of Fig. 7. This is a *nonphysical* region in which no one would ordinarily look. This discovery of the new solution provides the completeness needed in our discussions; the dispersion structure in Fig. 7 now has *three* solutions at each frequency, and the mystery in the number of solutions is removed. More important, this completeness aspect greatly helps to understand the migration of solutions in the *physical* region, relating to the simultaneous-propagation effect.

A very similar set of curves is found for slot line, as may be seen when we inspect the β/k_0 plot in Fig. 5, for the case for which the bound and leaky modes are separated, and Fig. 9, for the simultaneous-propagation case (the latter plot includes the improper solutions at lower frequencies, which are not contained in Fig. 6).

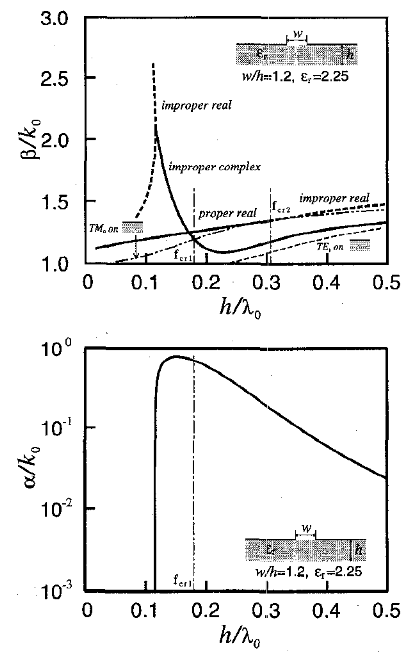

Fig. 9. A plot similar to that in Fig. 8, but for *wider* slot, $w/h = 1.20$, on *slot line*. The behavior for β/k_0 is seen to be similar to that in Fig. 8. A plot for α/k_0 is also included for completeness.

B. The Evolution of the Nonphysical and Physical Solutions

We next examine this new improper real solution and its role in the evolution from the dispersion behavior in Figs. 7 or 5 to that in Figs. 8 or 9. Since the evolution for the conductor-backed coplanar strips and that for the slot line are very similar, we will present results for only one of these lines, and we choose *slot line* because the spectral gap is wider and more pronounced for it, as mentioned in Section II-B. We can then see at each step, as we change the slot width, whether or not the spectral gap has been modified.

The new improper real solution, which is nonphysical everywhere, lies far above the other solutions shown in Fig. 5 in the β/k_0 plot. When we compress the vertical scale by a factor of two, however, we may observe this new improper real mode in the upper right corner of Fig. 10, which also holds for $w/h = 0.4$. The values of β/k_0 for this new mode are seen to be well above 2.0, which is clearly in a nonphysical range, being above $[\varepsilon_r]^{1/2}$. The points on the plot labeled A through D should be ignored for now; they correspond to vector electric field plots discussed in Section IV. As w/h is increased, the new solution drops down, and the other improper real solution, shown dotted, peaks upward to meet the new downward-coming solution, as seen in Fig. 11 for $w/h = 0.5$. At $w/h = 0.533$, as shown in Fig. 12, these two curves almost touch each other. The curve for α/k_0 has hardly changed as w/h has been increased from 0.4 to 0.533, except for a small shift in the onset of leakage toward higher frequencies. Also, the physical portions of the dispersion plots have changed negligibly, except for a small shift in the location of the spectral gap to higher frequencies.

As the w/h is increased only *slightly* further, to 0.535 from 0.533, the two curves touch and then pull away from each other in a direction at right angles to the original approach, produc-

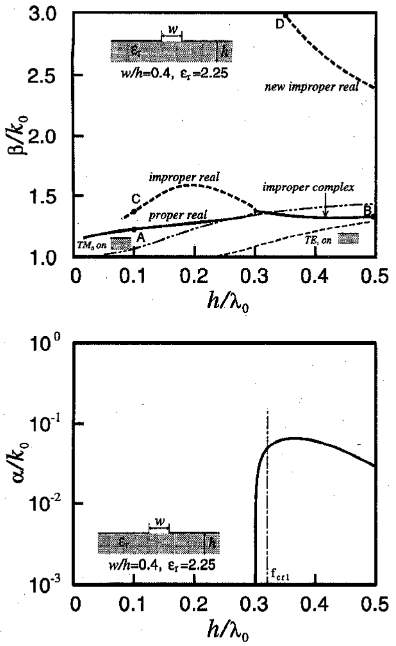

Fig. 10. A plot similar to that in Fig. 5 for β/k_0, but without the measured points, and with the ordinate scale compressed by a factor of 2. The new feature in this plot is the presence of the *new* improper real solution in the upper right corner.

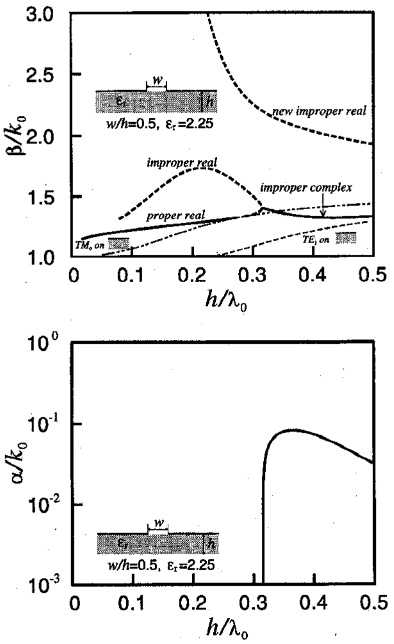

Fig. 11. A plot similar to that in Fig. 10, but for a somewhat wider slot, $w/h = 0.50$. The new improper real solution drops down, and the other (old) improper real solution rises to meet it.

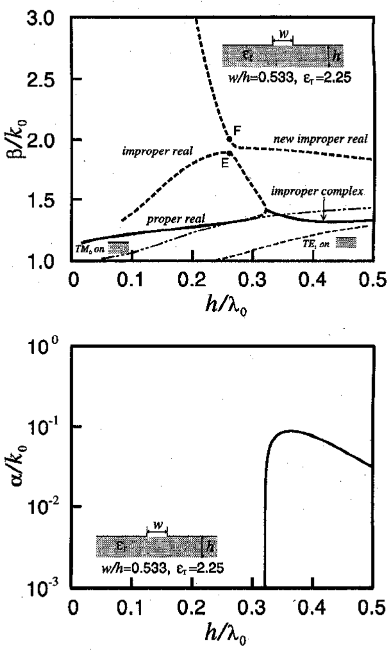

Fig. 12. A plot similar to that in Fig. 11, but for a slightly wider slot, $w/h = 0.533$. We see that now the new and old improper real solutions almost touch each other.

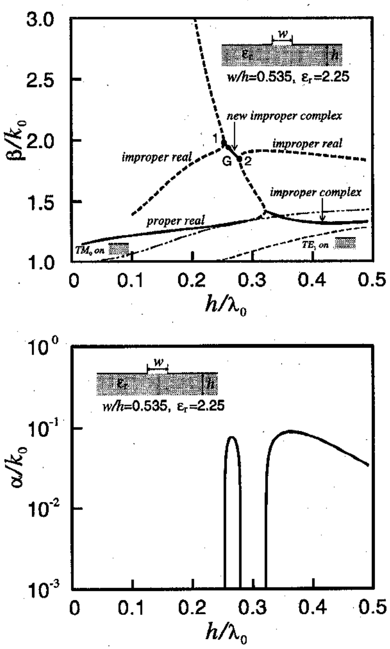

Fig. 13. A plot similar to that in Fig. 12, but for a *very slightly* wider slot, $w/h = 0.535$. The two improper real solutions have touched and then separated in a different fashion, with an improper complex solution in the gap between them. This new solution is leaky but nonphysical.

ing separate low-frequency and high-frequency portions. When they pull away, however, the gap between them is not empty but contains *a new improper complex (leaky) solution*, as seen in Fig. 13 between points 1 and 2. A new contribution is also present in the plot for α/k_0, but it is nonphysical. (The points labeled E and F in Fig. 12 and G in Fig. 13 refer to electric field plots in Section IV, and should therefore be ignored now.)

The unusual and highly sensitive behavior exhibited in Figs. 12 and 13 was also found to occur for other printed-circuit lines, and was reported for conductor-backed coplanar strips as Figs. 5 and 6 in the shortened version of this paper appearing in the 1995 International Symposium Digest.

The width of the interaction gap increases as w/h is increased further, accordingly, point 1 moves upward, to lower frequencies, while point 2 moves downward, to higher

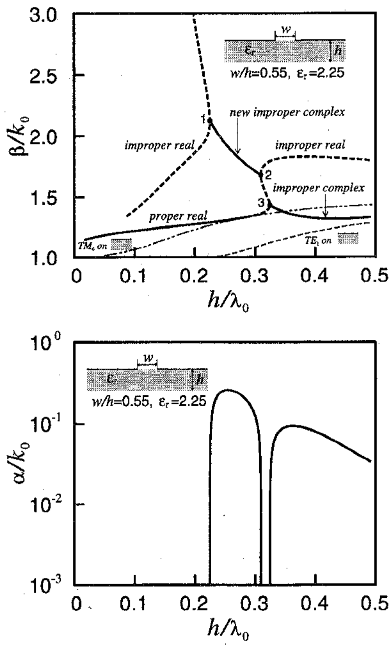

Fig. 14. A plot similar to that in Fig. 13, but for a still wider slot, $w/h = 0.55$. The gap containing the new improper complex mode is now much larger.

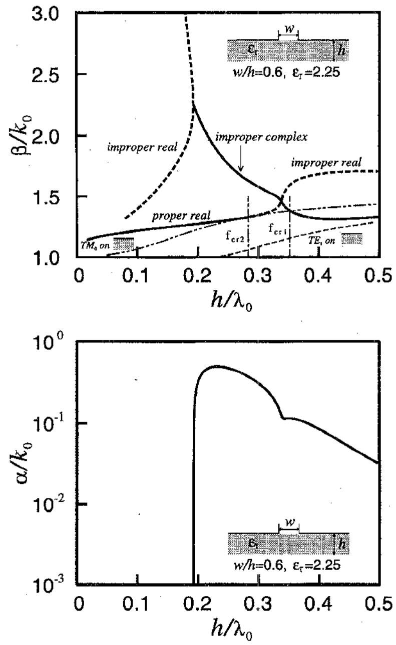

Fig. 15. A plot similar to that in Fig. 14, where the slot is made still wider, $w/h = 0.60$, with the result that the new improper complex solution and the original leaky-mode solution have become continuous with each other.

frequencies, as seen in Fig. 14, for $w/h = 0.55$. For still further increases in the w/h value, point 2 moves down until it meets point 3, at which time, when $w/h = 0.6$, the new complex (leaky) solution becomes *continuous* with the original leaky solution, as may be noted in Fig. 15. For that case, however, the spectral gap is still present, and there is no significant change yet in the dispersion behavior of the solutions in the physical region. As w/h is increased somewhat more, to 0.8, as in Fig. 16, the leaky solution moves down further and *crosses* the proper real solution at a lower frequency so that the spectral gap disappears, and the situation of simultaneous propagation results. The range in frequency for simultaneous propagation in Fig. 16 is still small, however, but it can be increased significantly by making the slot wider, as we found in Fig. 9, where $w/h = 1.2$.

Exactly analogous behavior is found for other printed-circuit transmission lines on both isotropic and anisotropic substrates, and also when the strips on coplanar strips are unequal in width.

Let us now summarize some interesting features related to the sequence of dispersion plots appearing in Figs. 10–16 and then 9, as a function of increasing slot width. As the normalized slot width w/h was varied from 0.4 to 0.6 (Figs. 10–15), the portion of dispersion plot corresponding to *physical* behavior changed negligibly, except for a slight shift in the location of the spectral gap. The bound and leaky modes remained completely separate from each other, and the spectral gap changed very little. If the *nonphysical* portions of these curves were omitted from the plots, we would see hardly any difference from one plot to the next one. Only for $w/h > 0.6$ can we find any changes in the physical portions; the spectral gap shrinks, and then disappears after the bound-mode and leaky-mode portions cross each other, which happens for

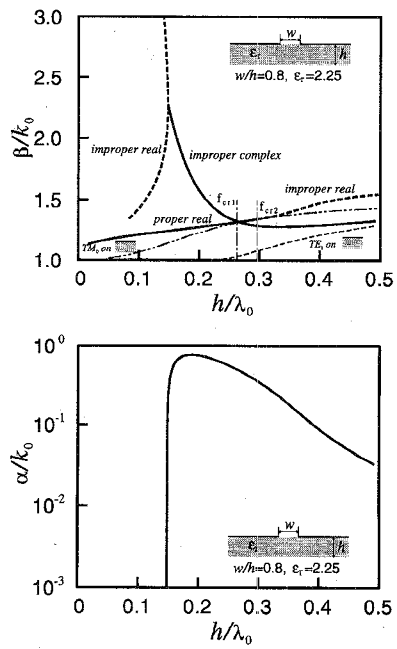

Fig. 16. A plot of the type of Fig. 15, but with a wider slot, $w/h = 0.80$. The leaky-mode solution has moved down and crossed the bound-mode solution, so that the spectral gap is no longer there and simultaneous propagation is now possible over a narrow frequency range, between f_{cr1} and f_{cr2}.

w/h somewhere between 0.7 and 0.8. After this crossing, an overlap region is formed within which the bound and leaky modes can propagate simultaneously. With respect to the *physical* solutions, therefore, we find no significant change when $w/h < 0.6$, but important and fundamental changes in character when $w/h > 0.6$ or so.

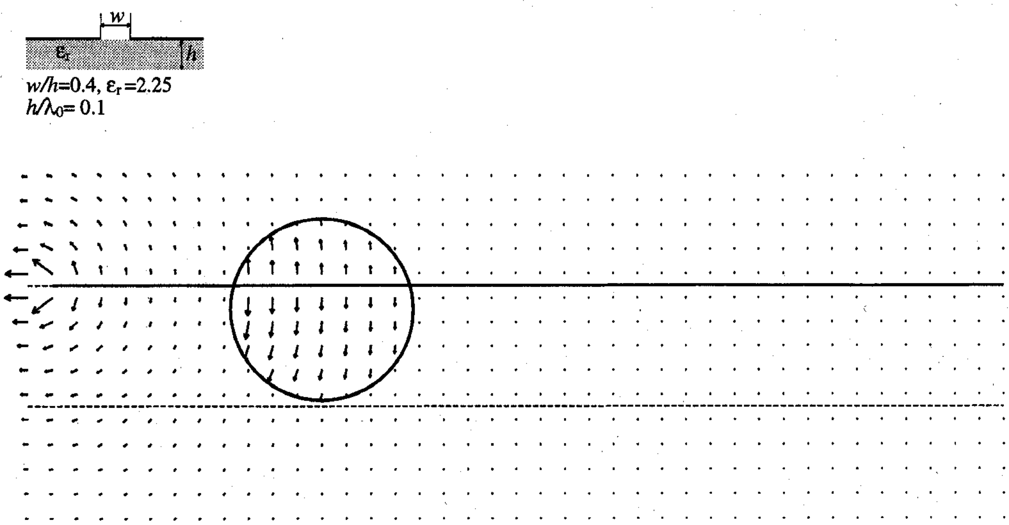

Fig. 17. Electrical field variation for the bound mode, corresponding to point A in Fig. 10.

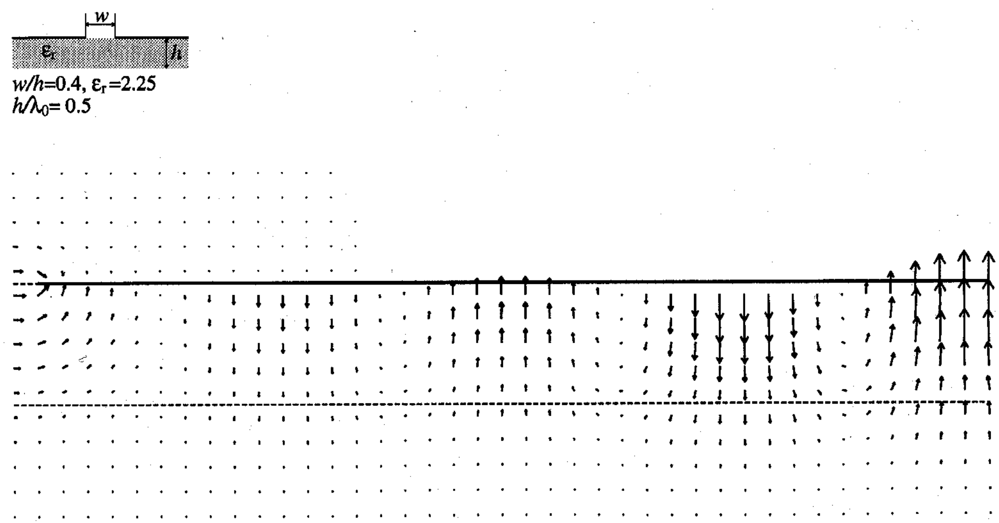

Fig. 18. Electrical field variation for the leaky mode, corresponding to point B in Fig. 10.

On the other hand, when w/h varies from 0.4 to 0.6 we observe from Figs. 10–15 that dramatic changes are occurring in the nonphysical solutions, particularly for w/h between 0.53 and 0.54. Furthermore, in the range of w/h for which large changes occur in the physical portions of the plots, the nonphysical solutions do change somewhat, but they retain their basic character. We therefore observe that the physical solutions change only slightly when the nonphysical solutions undergo dramatic changes in their primary features, and that, in reverse but not as strikingly, the nonphysical solutions merely shift around and change numerically, but not in a basic fashion, when the physical solutions change in a fundamental way, from separate bound and leaky modes to a frequency range in which these modes can propagate simultaneously.

Finally, we should recognize the key role played by the newly discovered improper real solution. Without that solution, and its interaction with the original improper real solution, we would not have been able to understand how the additional leaky (improper complex) solution in the nonphysical region could have originated, and in turn what could cause the fundamental changes in the physical solutions.

IV. Electric Field Variations for the Various Mode Types on Slot Line

As we scan the dispersion plots for β/k_0 in Figs. 10–16, we may observe that there are several different types of modal solutions. Four different modal solutions appear in Fig. 10 and are identified by the lettered points A, B, C, and D. Fig. 10 applies to the case of narrow slots in slot line, for which the bound and leaky modes, which are both physical and are identified by points A and B, respectively, are completely separated, with a spectral gap between them. An improper

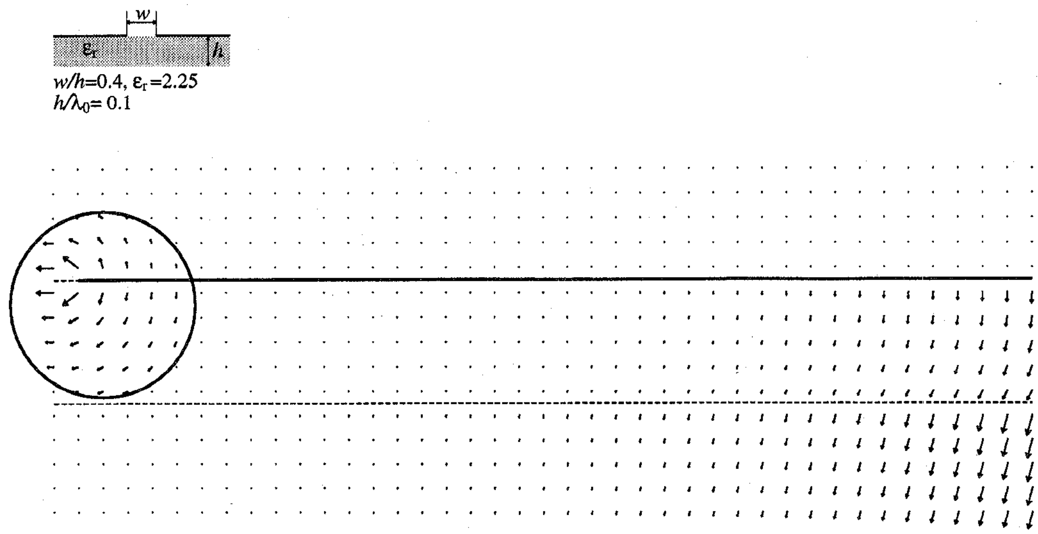

Fig. 19. Electrical field variation for the lower of the improper real solutions, which is nonphysical, and corresponds to point C in Fig. 10.

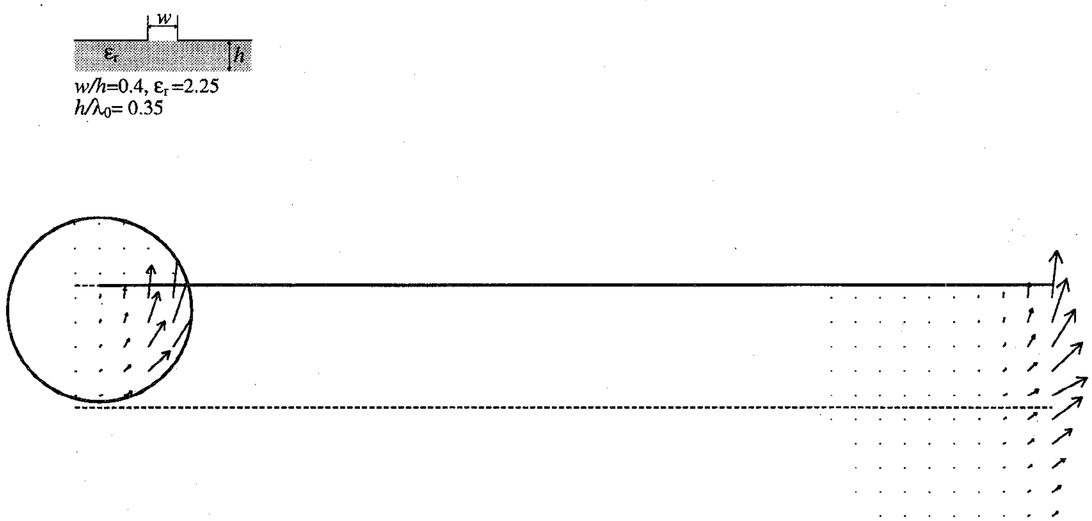

Fig. 20. Electrical field variation for the *new* improper real solution, corresponding to point D in Fig. 10. The unusual feature here is the enormous rate of transverse field increase.

real solution, which is nonphysical and identified by point C, emerges from the spectral gap and moves upward and back to lower frequencies. Finally, there is the new, previously unknown improper real solution, which is identified by point D and appears in the upper right corner.

Point A, located on the bound-mode (proper real) curve, should be characterized by an electric field variation in the transverse direction that is strongest near the center of the slot and then *decays* rapidly as one moves transversely away from the slot region. Such behavior, which is of course well known, is consistent with the "proper" nature of the solution, which requires that the field decays to zero in the transverse direction. The vector electric field variation corresponding to point A is presented in Fig. 17, where only the right half is shown. The solid horizontal line represents the metal plane, and the line with short dashes corresponds to the bottom of the dielectric substrate. The field in the circles is expanded by a factor of 8, and we may observe that the field is decreasing away from the slot region even within the circle.

Point B lies on the leaky-mode (improper complex) curve, and the electric field variation for it appears in Fig. 18. The "improper" nature of the leaky mode requires that the field *increases* transversely away from the strip, but it does so in a traveling-wave fashion because power is propagating away from the slot at an angle to the guide axis. The leaky mode is a physical mode because the field decreases in traveling-wave fashion along the slot, and the field never reaches infinity transversely. These aspects that characterize the leaky mode are also well known.

Point C is located on the curve for the lower of the improper real solutions, and the electric field variation for it is shown in Fig. 19. The field near the strip is similar to that for the bound mode, but, after decreasing at first, it *increases* transversely, as seen in Fig. 19, consistent with its "improper" nature. The

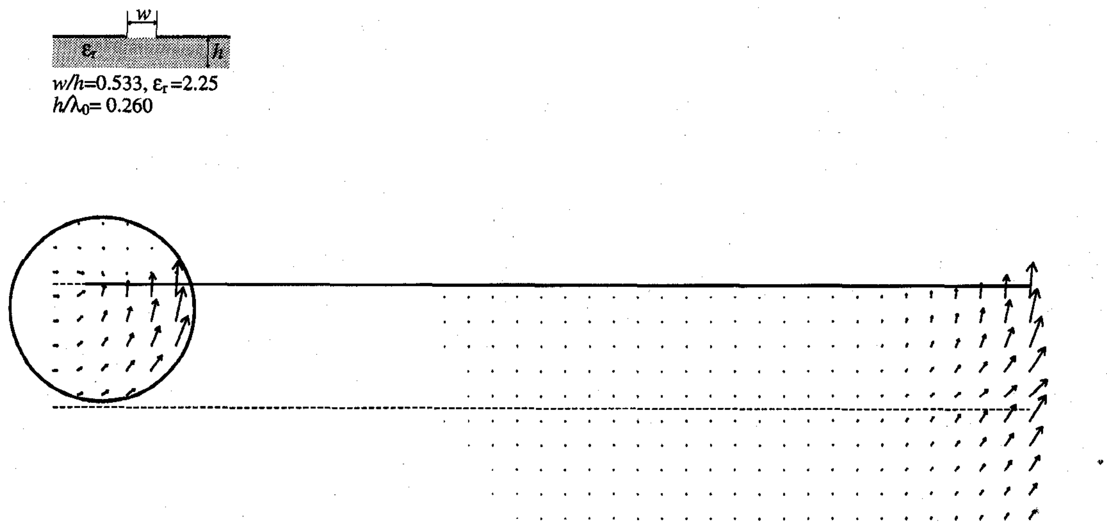

Fig. 21. Electrical field variation for the old improper real mode, corresponding to point E in Fig. 12. The behavior appears similar to that in Fig. 20, but with a much smaller rate of transverse field increase.

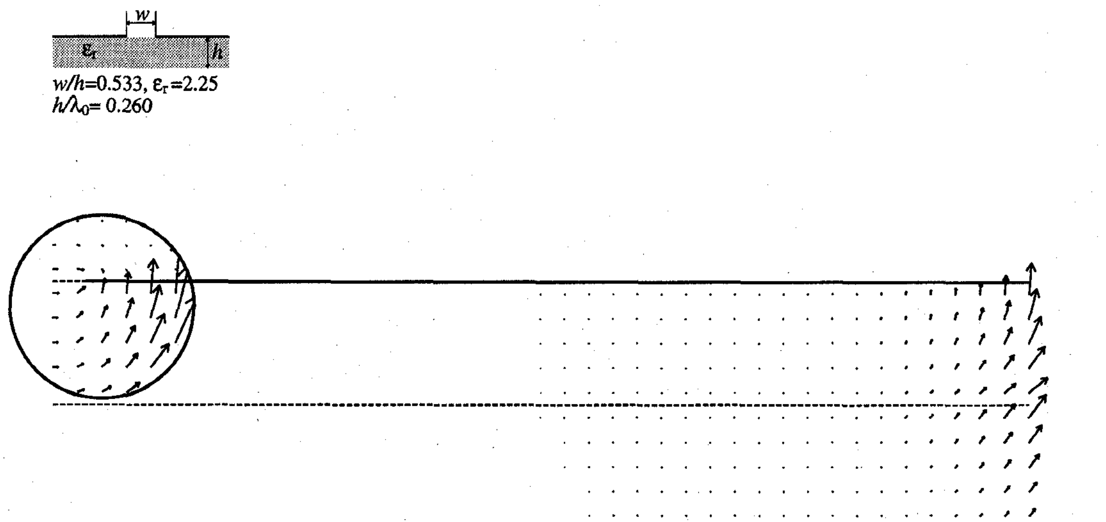

Fig. 22. Electrical field variation similar to that in Fig. 21, but for the new improper real mode corresponding to point F in Fig. 12.

increase is not associated with any power leakage, however, and is consistent with its "real" property, which means that the mode amplitude does not decrease as it propagates down the slot. Such a mode is nonphysical, since it would have to carry an infinite amount of power. Nevertheless, as discussed under Section III, the movements of the nonphysical solutions as the slot width changes explains the otherwise-mysterious transition from separate regions to simultaneous-propagation behavior for the physical solutions.

The last of the mode types is the *new* improper real solution identified by point D. The electric field variation at point D, as shown in Fig. 20, appears to be different in two ways from the variation seen in Fig. 19. First, the rate of transverse increase is seen to be *enormous*; we note that the fields inside the circle near the slot are expanded by a factor of 10^{18}. This very high rate of increase may surprise us until we actually put in the numbers. Second, the field in the vicinity of the slot does not appear to be tied to the slot, and does not seem to resemble the field in the slot that we find for the other three mode types. However, this apparent behavior may be the result of the astoundingly large rate of transverse increase.

Let us refer next to Fig. 12, where the old and the new improper real solutions approach each other closely and almost touch. Points E and F are at the same frequency but lie on the curves for the old and the new solutions, respectively. As seen in Figs. 21 and 22, for points E and F, the field variations for these two points are rather similar to each other, and also resemble the behavior in Fig. 20 but in a more subdued fashion since the enlargements in the circles are now given by factors of 10^6 and 10^8, rather than 10^{18}.

Lastly, we refer to point G on Fig. 13, which corresponds to the new improper complex, but nonphysical, solution formed

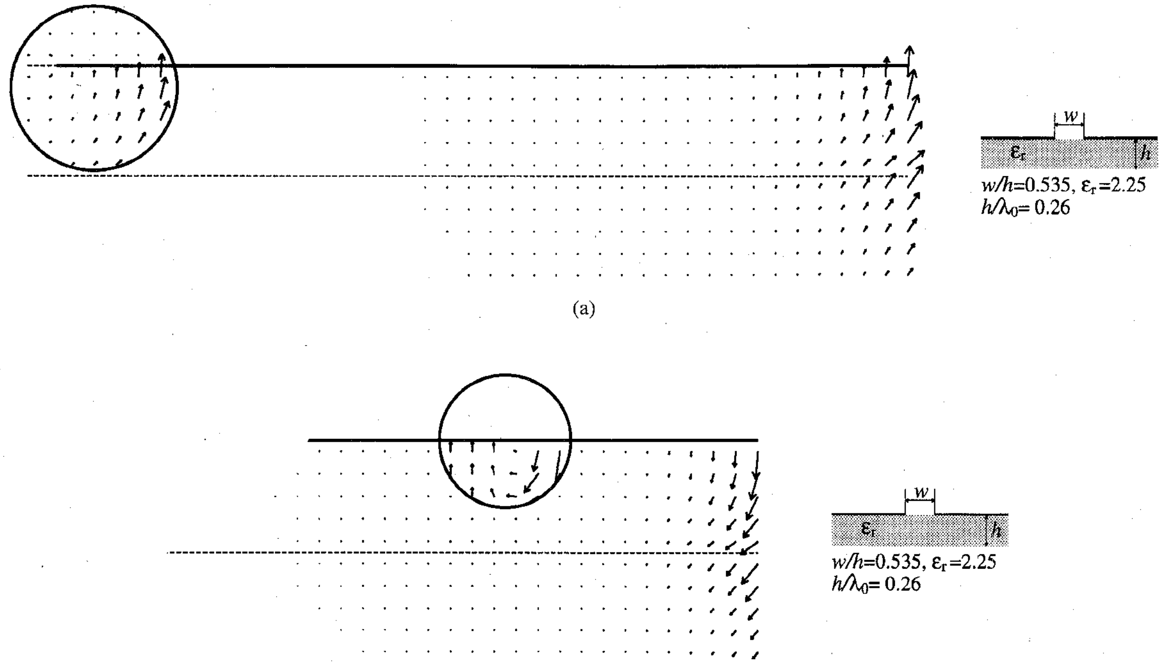

Fig. 23. Electrical field variation for the improper complex (leaky but nonphysical) mode, corresponding to point G on Fig. 13, which exhibits leaky-mode behavior but with a very high rate of transverse increase.

just after the two improper real solutions have separated from each other. The field variation, shown in Fig. 23, exhibits the leaky-mode behavior seen in Fig. 18 but influenced strongly by the high value of β/k_0, which produces a very high rate of transverse increase. We see that the field in the circle near the slot region has been expanded by 5×10^6, but we also may note that the field does demonstrate traveling-wave behavior, as may be seen within the second circle, which shows a sign reversal for the field.

The electric field variations in the transverse direction for the bound mode, leaky mode, and the original improper real mode, presented in Figs. 17–19, were all known to some degree from earlier studies and were expected. The new feature in these field plots involves the enormous rate of transverse field increase exhibited by the *new* improper real mode.

V. Conclusion

The effect we are discussing is that, by changing *only* the relative dimensions of a printed-circuit transmission line (such as a strip width or a slot width), the dispersion behavior for the dominant mode can change drastically from the expected case, for which the bound-mode and the leaky-mode regions are *completely separated* from each other, to an unexpected situation in which the bound and leaky modes can propagate *simultaneously* over a frequency range, which in fact can be very large. The practical importance of this result is that, if the transmission-line circuit is designed on the assumption that only the bound mode is present, but unexpectedly a leaky mode is there as well, the source designed to excite the bound mode will also excite the leaky mode, and with comparable amplitude because both modes have similar strip current distributions. The circuit will then suffer from unexpected cross talk and power loss.

When we originally found this simultaneous-propagation effect on conductor-backed coplanar strips, we believed that the phenomenon was a rarity. We have since discovered that this basic effect is quite *general*, and that it occurs on *most*, if not all, printed-circuit transmission lines, for both isotropic and anisotropic substrates. As examples, results are presented here for conductor-backed coplanar strips and for conventional slot line. Measurements were taken and are compared with the theoretical values to verify the reality of the new effect.

In this study, we also discovered the existence of a *new improper real mode*, which is *always nonphysical* but which serves to *explain* how the transition from separate regions to simultaneous propagation can occur. An examination of the evolution of this new solution as the relative line dimensions are modified shows that rather complicated changes occur in the nonphysical region and that the new solution plays a key role in them. It is particularly interesting that the complicated behavior in the nonphysical region ultimately leads to the important changes in the *physical* region discussed here.

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